Class-F Power Amplifiers with Maximally Flat Waveforms

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***Abstract*—Class-F power amplifiers (PA's) employ harmonic-frequency resonators to shape their drain or collector waveforms to improve efficiency. Generally, the output network must present the drain with either an open or short circuit at the harmonic frequencies. At VHF and higher frequencies, the drain capacitance, lead inductance, lead length, and dispersion make implementation of reasonably ideal tuned circuits difficult. However, it is possible to control the impedances at a finite number of harmonics. This note first derives the basic relationships among the Fourier coefficients of the waveforms and the performance of the amplifier. Fourier coefficients for maximally flat waveforms are then derived for inclusion of up to the fifth harmonic. Amplifier performance is then tabulated as a function of which harmonics are included in the voltage and current waveforms. Efficiency increases from 50% of class A toward 100% as harmonics are added. Power-output capability increases by up to 27%.**

***Index Terms*—Amplifier, class-F, power.**

I. Introduction

Class-F power amplifiers (PA's) use multiple-resonator output filters (Fig. 1) to control the harmonic content of their drain–voltage and/or drain–current waveforms. Flattening of the waveforms by the harmonics (Fig. 2) allows the majority of the drain current to flow when the drain voltage is low, reducing power dissipation. The efficiency of an ideal PA is increased from the 50% limit of class-A operation toward 100%, with the amount of increase depending upon which harmonics are controlled. In contrast to class C, the power-output capability can also be increased. Class F is probably the oldest technique for improving the efficiency of an RF PA, but is perhaps also the least well understood.

High-efficiency PA's are implemented with true switching-mode operation (classes D and E) at frequencies from VLF through lower VHF. The drain waveforms of these amplifiers are not sinusoidal and, therefore, contain harmonics. Even class-B operation requires the presence of harmonics to produce the half-sinusoid current waveforms. While the output network prevents the harmonics from reaching the load, it must allow their presence at its input. The presence of harmonics in turn requires the correct drain/collector impedance (open or shorted) at the harmonic frequency. If the output network does not permit any harmonic voltages or currents, only class-A operation (with its attendant low efficiency) is possible.

The benefits of flattening the bottom of the plate–voltage waveform were known as early as 1919, and Tyler is widely credited with the first implementation and general description of modern multiresonator class-F PA's in [1]. Prior to the availability of switching transistors suitable for class-D and class-E amplifiers [2], class-F with lumped-element resonators has been widely used in high-power A.M.-broadcast transmitters from LF through HF [3]–[6].

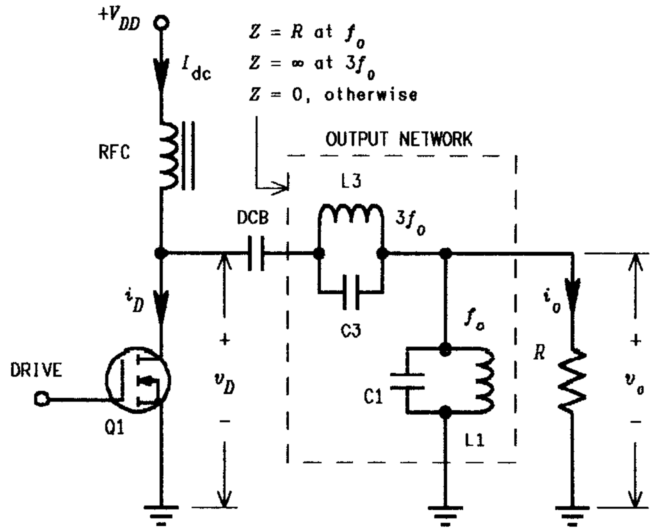

Fig. 1. Example (third-harmonic-peaking) class-F PA.

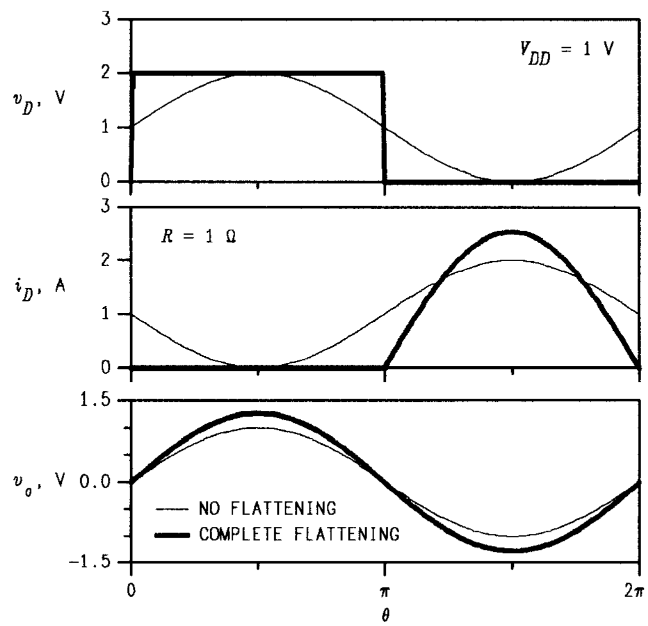

Fig. 2. Flattened and unflattened waveforms.

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Tyler [1] also notes the difficulty in implementing lumped-element multiresonator output networks at VHF and proposes the use of quarter-wavelength transmission lines that (ideally) control all harmonics. Class-F PA's with these networks are used in VHF-FM broadcast transmitters. The first application of class F to UHF PA's is generally credited to Snider [7], although his analysis deals primarily with an *optimally loaded* PA in which all harmonic impedances are either infinite or zero, as well as an overdriven PA in which harmonic power is delivered to the load.

At VHF and higher frequencies, it becomes increasingly difficult to find transistors capable of switching fast enough for class-D and class-E operation. In addition, the drain/collector capacitance, lead inductance, lead length (including bond wires), and dispersion (frequency-dependent propagation velocity) make implementation of reasonably ideal tuned circuits difficult. Since the output networks cannot support the required waveforms, ideal class-B, -D, or -E operation is not possible. Fortunately, it is often possible to control the impedances at a finite number of harmonics, which allows class-F PA's to achieve efficiencies considerably better than that of class A.

The current interest in cellular and personal communication at UHF has spurred a plethora of developments using class F. Most of these use transmission-line networks to control the impedance at the second [8], [9], or third harmonic [10]. A number use transmission lines to control both the second and third harmonics [11], [12]. Toyoda uses dielectric resonators to implement classical Tyler-type third- and fifth- harmonic-peaking PA's [13] at 900 MHz. Others demonstrate the effect of harmonic terminations through experiment and simulation [14], [15].

The characteristics of a PA can be derived from the frequency components of its voltage and current waveforms through theory developed by Molnar and Kazimierczuk [15]–[17]. This note uses these techniques to derive the characteristics of class-F PA's with various harmonic impedances. These results can be used by design engineers to assess the benefits of load networks of various complexities.

II. Theory

The output network of a generic class-F PA is assumed to be an ideal *LC* or transmission-line filter (linear, passive, and lossless) that allows only fundamental-frequency power to pass through to the load. The active device (MOSFET, BJT, MESFET, etc.) is assumed to be an ideal current source (or in certain cases, an ideal switch), with neither on-state resistance, saturation voltage, nor output capacitance.

Since no harmonic power can reach the load, and the output network is lossless, no harmonic power can be generated by the transistor. In a class-F PA, this is accomplished by the load network producing either zero or infinite impedances at all frequencies at which harmonics are generated. As a result, either voltage or current, but not both, are present at any given harmonic frequency.

A. Waveforms

Flattening can be accomplished by any harmonic or combination of harmonics. Harmonics of lower order are more effective (as subsequently shown) than those of higher order, hence, lowest order harmonics are added first. The use of both even and odd harmonics in the same waveform causes it to approximate a narrow (class-C current) pulse, which increases efficiency at the cost of power-output capability. Improvement of both efficiency and power-output capability is best accomplished by using odd harmonics to cause one waveform to approximate a square-wave, and even harmonics to cause the other to approximate a half-sinusoid. The subsequent analyses are based upon the drain voltage and current waveforms

$$\nu_D(\theta) = V_{DD} + V_{om}\sin\theta + V_{3m}\sin 3\theta + V_{5m}\sin 5\theta +, \cdots \tag{1}$$

and

$$i_D(\theta) = I_{\mathrm{dc}} - I_{om}\sin\theta - I_{2m}\cos 2\theta - I_{4m}\cos 4\theta +, \cdots \tag{2}$$

where $\theta = \omega t$ and ω is the (fundamental) frequency of the desired output. Alternately, the voltage waveform can contain even harmonics and the current waveform can contain odd harmonics.

B. Output Power and Efficiency

The basic waveform parameters γ_V, γ_I, δ_V, and δ_I relate the dc component to the fundamental-frequency component and peak of a waveform. For the drain voltage

$$V_{om} = \gamma_V V_{DD} \tag{3}$$

and

$$\nu_{D\max} = \delta_V V_{DD}. \tag{4}$$

Similarly, for the drain current

$$I_{om} = \gamma_I I_{\mathrm{dc}} \tag{5}$$

and

$$i_{D\max} = \delta_I I_{\mathrm{dc}}. \tag{6}$$

The load network presents the drain with impedance $Z(f) = R$ at the fundamental frequency. The output voltage and current are, therefore, related by

$$V_{om} = I_{om} R \tag{7}$$

and the output power is

$$P_o = \frac{V_{om}^2}{2R} = \frac{\gamma_V^2 V_{DD}^2}{2R}. \tag{8}$$

The dc-input current is

$$I_{\mathrm{dc}} = \frac{I_{om}}{\gamma_I} = \frac{V_{om}}{R\gamma_I} = \frac{\gamma_V V_{DD}}{R\gamma_I}. \tag{9}$$

Hence, the dc-input power is

$$P_i = V_{DD} I_{\mathrm{dc}} = \frac{\gamma_V V_{DD}^2}{\gamma_i R} \tag{10}$$

and the efficiency is then

$$\eta = \frac{P_o}{P_i} = \frac{\gamma_V \gamma_I}{2}. \tag{11}$$

TABLE I
PARAMETERS OF ODD MAXIMALLY FLAT WAVEFORMS

n	δ_V	$\gamma_V = V_{om}/V_{DD}$	V_{3m}/V_{DD}	V_{5m}/V_{DD}
1	2	1	-----	-----
3	2	9/8 = 1.125	1/8 = 0.125	-----
5	2	75/64 = 1.172	25/64 = 0.391	15/64 = 0.235
∞	2	4/π = 1.273	4/3π = 0.424	4/5π = 0.255

NOTE: n = highest odd harmonic present.

C. Power-Output Capability

The power-output capability (output power with $\nu_{D\max} = 1$ V and $i_{D\max} = 1$ A) is obtained by dividing (8) by (4) and (6), which yields

$$P_{\max} = \frac{P_o}{\nu_{D\max} i_{D\max}} = \frac{\gamma_V \gamma_I}{2\delta_V \delta_I} = \frac{\eta}{\delta_V \delta_I}. \qquad (12)$$

This expression is further simplified at the end of Section III.

III. MAXIMALLY FLAT WAVEFORMS

The Fourier coefficients of a maximally flat waveform are obtained by adjusting their values so that the derivatives of the waveform are zero at the maximum and/or minimum voltage or current. These waveforms are both mathematically tractable and a good approximation to the waveforms which occur in a real class-F PA.

A. Voltage (Odd) Waveform

Maximally flat voltage waveforms are shown in Fig. 3, and their parameters are summarized in Table I. The voltage waveform (1) reaches its maximum and minimum values at $\theta = \pi/2$ and $3\pi/2$, respectively. Since $\cos(\pi/2) = \cos(3\pi/2) = 0$, the odd-order derivatives are inherently zero at these points. Maximum flatness at the minimum voltage requires the even derivatives to be zero at $\theta = 3\pi/2$.

The second and fourth derivatives of the voltage waveform (1) are

$$\frac{d^2\nu_D}{d\theta^2} = -V_{om}\sin\theta - 9V_{3m}\sin 3\theta - 25V_{5m}\sin 5\theta \qquad (13)$$

and

$$\frac{d^4\nu_D}{d\theta^4} = V_{om}\sin\theta + 81V_{3m}\sin 3\theta + 625V_{5m}\sin 5\theta. \qquad (14)$$

Substitution of $\theta = 3\pi/2$ into (13) and (14) yields

$$0 = V_{om} - 9V_{3m} + 25V_{5m} \qquad (15)$$

and

$$0 = -V_{om} + 81V_{3m} - 625V_{5m}. \qquad (16)$$

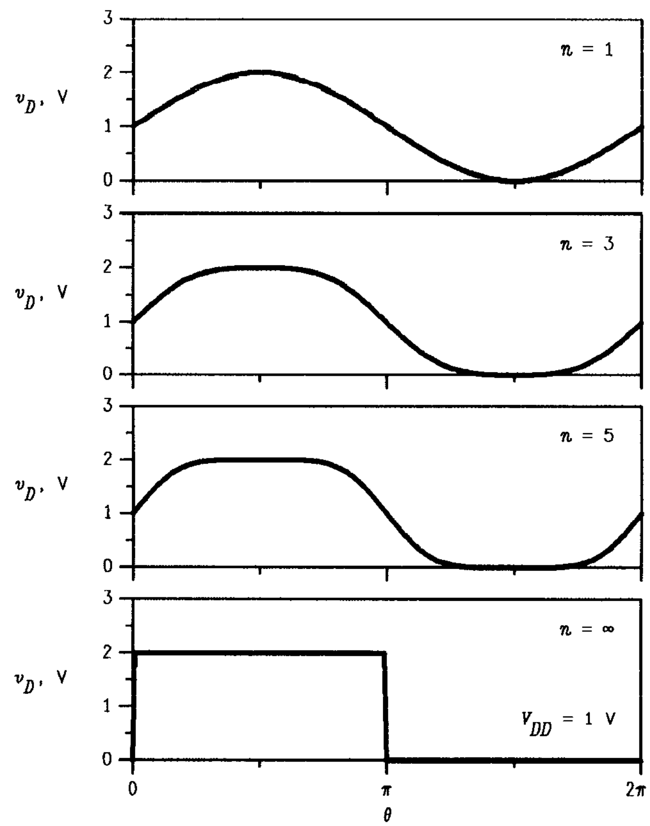

Fig. 3. Voltage (odd-harmonic) waveforms.

1) Third-Harmonic Peaking: If only the third harmonic is present, $V_{5m} = 0$. Equation (15) then yields $V_{3m} = (1/9)V_{om}$. For drain voltage (1) to reach zero at $\theta = 3\pi/2$:

$$0 = V_{DD} - V_{om} + V_{3m}. \qquad (17)$$

Hence,

$$V_{om} = (9/8)V_{DD} \qquad (18)$$

from which $\gamma_V = (9/8)$ and $V_{3m} = (1/8)V_{DD}$. The peak drain voltage is then

$$\nu_{D\max} = V_{DD} + V_{om} - V_{3m} = 2V_{DD} \qquad (19)$$

from which $\delta_V = 2$.

TABLE II
PARAMETERS OF EVEN MAXIMALLY FLAT WAVEFORMS

m	δ_I	$\gamma_I = I_{om}/I_{dc}$	I_{2m}/I_{dc}	I_{4m}/I_{dc}
1	2	1	-----	-----
2	8/3 = 2.667	4/3 = 1.333	1/3 = 0.333	-----
4	128/45 = 2.844	64/45 = 1.422	16/45 = 0.356	1/45 = 0.022
∞	π = 3.142	π/2 = 1.571	2/3 = 0.667	2/15 = 0.133

NOTE: m = highest even harmonic present.

2) Fifth-Harmonic Peaking: Simultaneous solution of (15) and (16) for V_{3m} and V_{5m} yields $V_{3m} = (1/6)V_{om}$ and $V_{5m} = (1/50)V_{om}$. For drain voltage (1) to reach zero at $\theta = 3\pi/2$:

$$0 = V_{DD} - V_{om} + V_{3m} - V_{5m}. \tag{20}$$

Hence,

$$V_{om} = (75/64)V_{DD} \tag{21}$$

from which $\gamma_V = 75/64, V_{3m} = (25/128)V_{DD}$ and $V_{5m} = (3/128)V_{DD}$.

The peak drain voltage is then

$$\nu_{D\max} = V_{DD} + V_{om} - V_{3m} + V_{5m} = 2V_{DD} \tag{22}$$

from which $\delta_V = 2$. Since the odd harmonics raise the voltage at $\theta = \pi/2$ by the same amount it is lowered at $3\pi/2$, the peak voltage is always $2V_{DD}$.

3) Square Wave: When all odd harmonics are present, the maximally flat waveform is a square wave. Simple Fourier analysis [2] yields

$$V_{om} = (4/\pi)V_{DD} \tag{23}$$

and, of course, $\nu_{D\max} = 2V_{DD}$.

B. Current (Even) Waveform

Maximally flat current waveforms are shown in Fig. 4, and their parameters are summarized in Table II. Inspection of Fig. 4 shows that the presence of even harmonics accentuates the peak at $\theta = 3\pi/2$ and flattens the bottom at $\theta = \pi/2$. The odd-order derivatives are again inherently zero at the minimum voltage. Therefore, maximum flatness requires that the even derivatives be zero at $\theta = \pi/2$.

The second and fourth derivatives of the current waveform (2) are

$$\frac{d^2 i_D}{d\theta^2} = I_{om}\sin\theta + 4I_{2m}\cos 2\theta + 16I_{4m}\cos 4\theta \tag{24}$$

and

$$\frac{d^4 i_D}{d\theta^4} = -I_{om}\sin\theta - 16I_{2m}\cos 2\theta - 256I_{4m}\cos 4\theta. \tag{25}$$

From (24) and (25),

$$0 = I_{om} - 4I_{2m} + 16I_{4m} \tag{26}$$

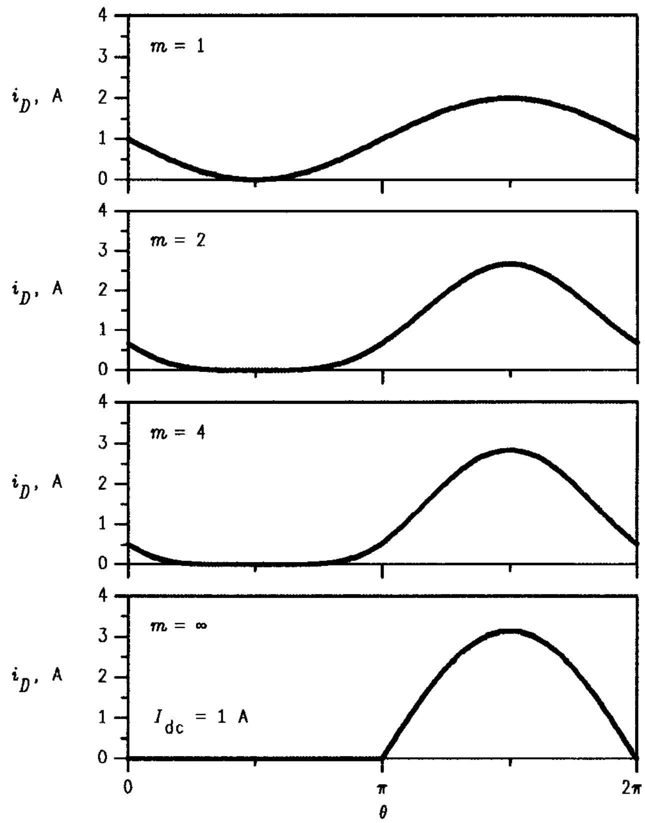

Fig. 4. Current (even-harmonic) waveforms.

and

$$0 = -I_{om} + 16I_{2m} - 256I_{4m}. \tag{27}$$

The minimum current is

$$i_{D\min} = I_{dc} - I_{om} + I_{2m} - I_{4m} \tag{28}$$

and the maximum current (at $\theta = 3\pi/2$) is

$$i_{D\max} = I_{dc} + I_{om} + I_{2m} - I_{4m}. \tag{29}$$

1) Second-Harmonic Peaking: If only the second harmonic is present, $I_{4m} = 0$. From (26), $I_{2m} = (1/4)I_{om}$. For the minimum current to be zero, (28) implies that

$$I_{om} = (4/3)I_{dc} \tag{30}$$

from which $\gamma_I = 4/3$ and $I_{2m} = (1/3)I_{dc}$. Insertion of these values into (29) yields

$$i_{D\max} = (8/3)I_{dc} \tag{31}$$

from which $\delta_I = 8/3$.

TABLE III
PERFORMANCE OF CLASS-F PA's FOR VARIOUS COMBINATIONS OF HARMONICS

m	η: $n = 1$	$n = 3$	$n = 5$	$n = \infty$
1	1/2 = 0.500 Class A	9/16 = 0.563	75/128 = 0.586	2/π = 0.637
2	2/3 = 0.667	3/4 = 0.750 HRA [11]	25/32 = 0.781	8/3π = 0.849 2nd HP
4	32/45 = 0.711	4/5 = 0.800	5/6 = 0.833	128/45π = 0.905
∞	π/4 = 0.785 Class B	9π/32 = 0.884 3rd HP	75π/256 = 0.920 3+5 HP [13]	1 = 1.000 Class D, F-TL

	P_{max}: $n = 1$	$n = 3$	$n = 5$	$n = \infty$
	1/8 = 0.125	9/64 = 0.141	75/512 = 0.146	1/2π = 0.159

2) Fourth-Harmonic Peaking: Simultaneous solution of (26) and (27) yields $I_{2m} = (5/16)I_{om}$ and $I_{4m} = (1/64)I_{om}$. For $i_{D\min} = 0$, (28) requires that

$$I_{om} = (64/45)I_{\rm dc} \tag{32}$$

from which $\gamma_I = 64/45$, $I_{2m} = (4/9)I_{\rm dc}$, and $I_{4m} = (1/45)I_{\rm dc}$. Insertion of these values into (29) yields

$$i_{D\max} = (128/45)I_{\rm dc} \tag{33}$$

from which $\delta_I = 128/45$.

3) Half-Sine Wave: When all even harmonics are present, the maximally flat current waveform is a half-sinusoid. Its fundamental-frequency component is

$$I_{om} = (\pi/2)I_{\rm dc} \tag{34}$$

and its peak is

$$i_{D\max} = \pi I_{\rm dc}. \tag{35}$$

C. Power-Output Capability

Subtraction of (28) from (29) yields

$$i_{D\max} - i_{D\min} = 2I_{om}. \tag{36}$$

Since $i_{D\min} = 0$,

$$i_{D\max} = 2I_{om} = 2\gamma_I I_{\rm dc}. \tag{37}$$

As a result,

$$\delta_I = 2\gamma_I \tag{38}$$

for an even waveform. Substitution of this and $\delta_V = 2$ (for the odd waveform) into (12) yields

$$P_{\max} = \frac{\gamma_V}{8}. \tag{39}$$

Therefore, the power-output capability depends only upon the ratio of the fundamental-frequency and dc components of the *odd* waveform.

IV. PA PERFORMANCE

The performance of class-F PA's with various combinations of voltage and current harmonics is predicted by inserting values of the waveform parameters (Tables I and II) into (11), (12), or (39). The resultant efficiencies and power-output capabilities (Table III) agree with the well-known characteristics of class A, class B, third-harmonic peaking, second-harmonic peaking, and class F with quarter-wavelength transmission lines.

The reader should remember that the roles of voltage and current can be reversed; i.e., the voltage waveform can be composed of even harmonics and the current waveform composed of odd harmonics. This is the case for second-harmonic peaking and the Kazimierczuk class-F PA, which are duals of those of Table III. Their analysis requires only that the roles of waveform parameters be interchanged (i.e., γ_I and δ_I are used in place of γ_V and δ_V, and vice versa).

Table III shows that the efficiency increases when the number of harmonics in either the voltage or current is increased. It is also clear that there is more benefit to including harmonic resonators for the appropriate waveform in numerical order (i.e., 2, 3, 4, $\cdots$,) rather than trying to increase the number of harmonics in only one waveform or another.

The somewhat popular class-F configuration with second- and third- harmonic resonators has an efficiency of 0.75, which is roughly that of class B. Inclusion of all harmonics through the fifth increases the efficiency to 0.833. As previously noted, power-output capability is a function of the number of odd harmonics and increases from 0.125 for class B to the 0.159 of class D when all odd harmonics are present (and therefore, the waveform is square).

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